Quantum Mechanics (Q.M.)

Why Quantum Mechanics ?

⇒ Because Classical mechanics describes the motion of the particles in macroscopic scale only. Classical mechanics (C.M.) cannot explains the following mentioned;

i). The motion of electrons, protons, etc.

ii). Stability of atoms and discrete spectra of atoms.

iii). Phenomena like photoelectric effect, compton's effect, Raman effect.

iv). Black body radiation, etc.

Quantum Mechanics describes the motion & interaction of particles at the microscopic scale.

1). Light is a wave and can act as a Particle, having energy

$$E = h\nu \quad \text{— (1)}$$

where, E = Energy of a photon
h = planck's constant = 6.63×10^{-34} Js.
ν = frequency in Hz.

Eqn (1) is known as Max-Planck hypothesis (1900).

2). de-Broglie's hypothesis (1924):

It says that all matter has both particle and wave nature. The wave nature of a particle is given by

$$\lambda_d = \frac{h}{p} \quad \text{— (2)}$$

where, λ_d = de-broglie wavelength
h = planck's constant
p = momentum of the particle.

Ex: $e^{\ominus}$, proton, etc.

3). Heisenberg Uncertainity Principle (1927):

The more accurately you know the position (Δx), the less accurately you know the momentum (Δp) and vice-versa, i.e;

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi} = \frac{\hbar}{2} \quad \text{— (3)}$$

where, $\hbar = \frac{h}{2\pi}$

de-Broglie wave eqn :

Let us consider a photon as a wave, ~~then~~ having frequency 'ν', the energy is

$$\boxed{E = h\nu} \quad (1)$$

and when it is considered as particle of mass 'm', then it's energy is

$$\boxed{E = mc^2} \quad (2)$$

Equating eqn (1) & (2), we have

i.e, $h\nu = mc^2 \Rightarrow h\frac{c}{\lambda} = mc^2$ $\left[\because \nu = \frac{c}{\lambda}\right]$ [and $m.c = p$]

$\Rightarrow \frac{hc}{\lambda} = p.c \Rightarrow \lambda . p = h$

$$\Rightarrow \boxed{\lambda = \frac{h}{p}} \quad (3)$$

{$\because$ Photon has no rest mass ($m_0 = 0$) but has a momentum, $p = m.c$}

Note :

1). For massless particle like photon, the momentum, $p = m.c$.

$$\text{i.e. } \boxed{\lambda = \frac{h}{m.c}} \quad (4)$$

2). Whereas, for particle of mass 'm', moving with velocity 'v', the momentum, $p = m.v$

$$\text{i.e. } \boxed{\lambda = \frac{h}{m.v}} \quad (5)$$

3). When, $v << c$: $K.E. = \frac{1}{2}mv^2 \Rightarrow mv^2 = 2\,K.E.$

$\Rightarrow m^2v^2 = 2\,K.E. \cdot m \Rightarrow p^2 = 2\,m.K.E \Rightarrow \boxed{p = \sqrt{2m\,K.E.}} \quad (6)$

where, K.E = Kinetic energy

$\therefore$ Eqn (3) becomes, $\boxed{\lambda = \frac{h}{\sqrt{2m\,K.E.}}} \quad (7)$

where, λ = de-broglie's wavelength.

4). Particles with charge 'q' accelerated through a potential difference 'V' volts, ~~posses~~ posses a K.E., given by $K.E = qV$.

$\therefore$ Eqn (3) becomes, $\boxed{\lambda = \frac{h}{\sqrt{2mqV}}} \quad (8)$

Properties of de-Broglie's matter waves :

1. The wavelength associated with de-Broglie's matter waves is given by

$$\boxed{\lambda_m = \frac{h}{mv}} \quad \text{—} (1)$$

2. If $m \to 0 \Rightarrow \lambda_m \to \infty$ (→ tends to) i.e. λ_m cannot be calculated as it is very large.
3. If $m \to \infty \Rightarrow \lambda_m \to 0$ i.e. λ_m cannot be calculated as it is very very small.
4. If particle is at rest $v = 0$, then $\lambda_m \to$ Undefined (Very large)
5. If particle is moving with $v = c$, then $\lambda_m \to$ small

(In short, from 2. to 5., if mass is large, λ_m will be small. Similarly, if velocity is large, λ_m will be small.)

6. Matter waves are neither electromagnetic nor acoustic wave, but just a hypothetical wave. (May or may not be produced by charged particles)
7. Matter waves velocity depends on the wavelength, whereas e.m. waves travel with same velocity, independent of the wavelength.
8. These are also known as probability waves. The wave nature of matter introduces an uncertainity in the position of particle. The amplitude of the wave reveals the probability of finding the particle in space at a particular time. A larger amplitude means larger probability at that position.
9. It exhibits diffraction phenomena as any other waves.
10. Also known as pilot wave, as matter wave for moving particle is represented as wave packet (group of waves).

Example :